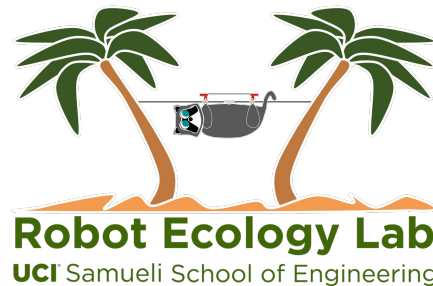


Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule

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Motivation



Los Angeles County fires: Maps of where the wildfires are [1]

Observations:

- 4 Distinct Regions of Interest
- Each has a certain fire intensity
- Each has a certain population density

Multi-Robotic Solution?

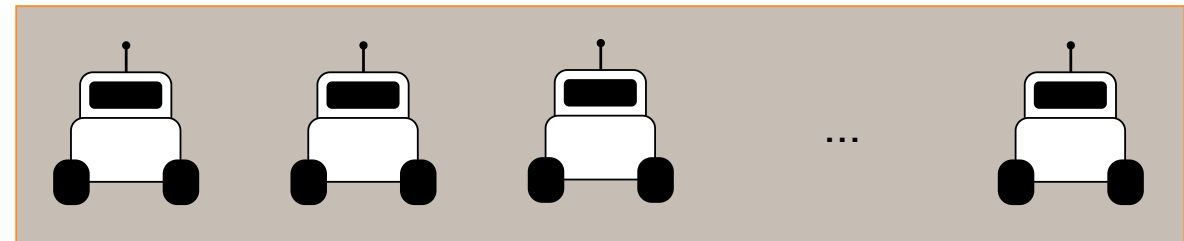
Previous Literature & Resource Allocation



Dynamic Area Coverage with Fire-Fighting Robots [1]



? Resource Allocation ?



Resource Allocation in Multi-Agent Systems, e.g., [2]

Q1. How to implement resource allocation for such scenarios?

[1]: R. Lin, S. Kim, and M. Egerstedt, "Heterogeneous Collaborative Pursuit via Coverage Control Driven by Fokker-Planck Equations," IEEE Transactions on Robotics (T-RO), 2025.

[2]: P. Anussornnitisarn, S. Y. Nof, and O. Etzion, "Decentralized control of cooperative and autonomous agents for solving the distributed resource allocation problem," International Journal of Production Economics, vol. 98, no. 2, pp. 114–128, 2005.

Ecological Inspiration

Hamilton's Rule of Altruism: [1]

$$rB \geq C$$

- r : genetic relatedness
- B : benefit to the recipient
- C : cost to the donor



Lioness adopts deceased sister's cubs [2]

- Genetic relatedness: sibling relationship, lioness and her deceased sister
- Benefit to the lion cubs' deceased mother: her genes will survive through her cubs' survival
- Cost to the lioness: sacrificing her lion cubs' resources and splitting her care.

Q2. How to translate Hamilton's rule to robotic systems?

Lioness

Team of
Robots

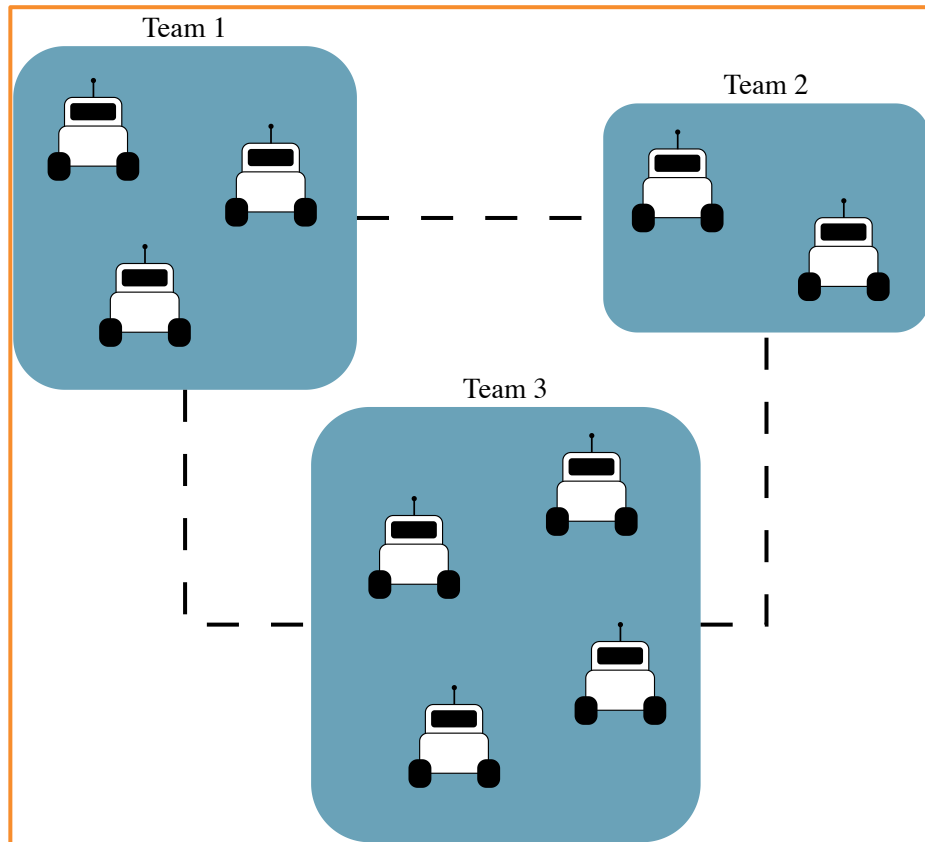
Genes

Mission
Weights

[1]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17-52, 1964.

[2]: <https://www.dailymail.co.uk/news/article-4418844/Lioness-suckles-orphaned-cubs-adopted-sister.html>

Collaboration: Robots as Resources



Graph Modeling: [1]

- Let there be m teams of robots
- $G = (V, E)$: undirected graph
- $V = \{v_1, \dots, v_m\}$: set of **teams**
- $E \subseteq V \times V$: set of **potential collaborative interactions**

Key Idea:

Compute potential collaborative interactions inspired by Hamilton's Rule [2]

Assumption 1:

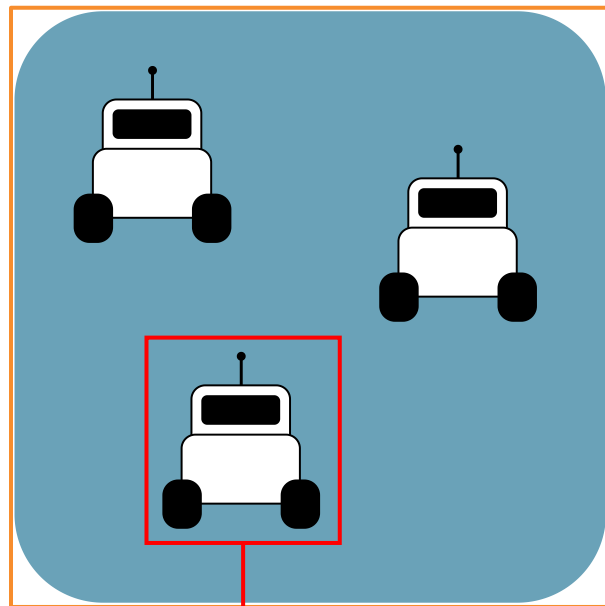
All agents in all m teams are homogenous, i.e., identical

[1]: M. Mesbahi and M. Egerstedt, Graph Theoretic Methods in Multiagent Networks. Princeton University Press, 2010.

[2]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17–52, 1964.

R. Karam, R. Lin, B.A. Butler, and M. Egerstedt, "Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule", submitted to the 2025 IEEE Conference on Decision and Control (CDC).

Team-Level Collaboration Framework



Robot = Resource

How to define a team's mission performance?



Mission Evaluation Function

Which parameter(s) to consider?



Number of Robots in a Team



Mission Evaluation Function for team k : $F_k(n_k)$,
with n_k : number of robots in team k

Team-Level Collaboration Framework

Mission Evaluation Function for team k : $F_k(n_k)$, with n_k : number of robots in team k

Assumption 2:

For each team k , the mission evaluation function $F_k(n_k) : \mathbb{N} \rightarrow \mathbb{R}$ satisfies the following properties:

1. **Strictly Increasing:** For all $n \geq 1$,

$$F_k(n+1) > F_k(n)$$

Higher is better

2. **Diminishing Returns:** For all $n \geq 2$,

$$F_k(n+1) - F_k(n) \leq F_k(n) - F_k(n-1)$$

Discrete Concavity

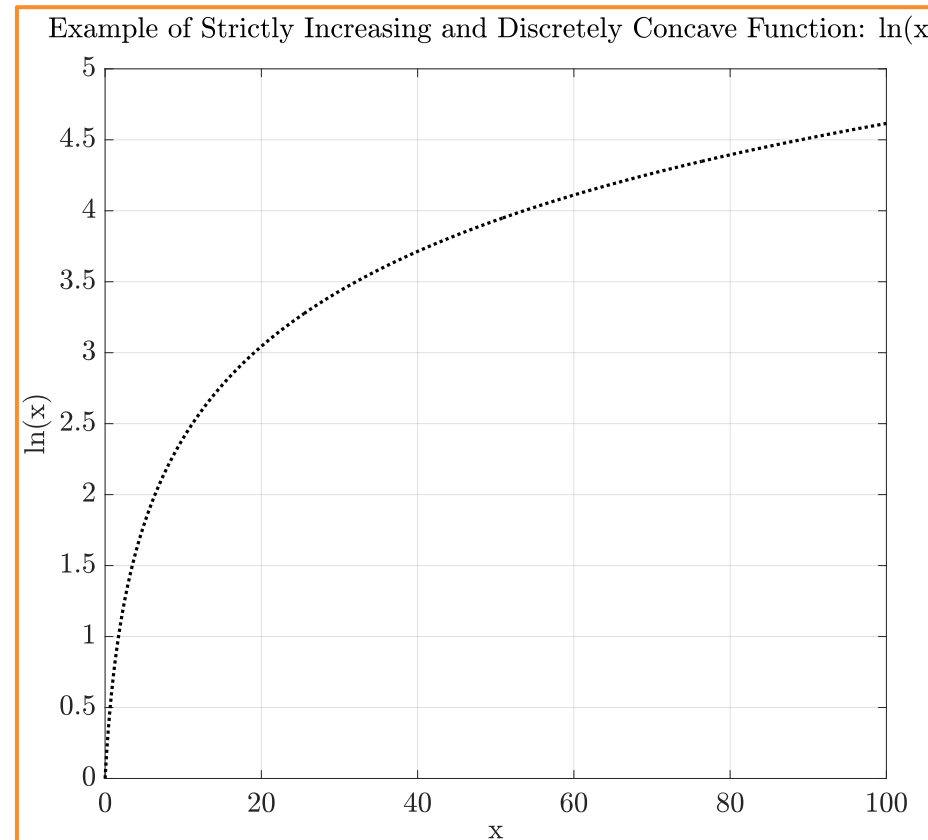
Scenario: [1]

There won't be overcrowding of agents and the number of agents in each team at any point in time won't reach a point where decreasing returns occur.

[1]: J. Ferber and G. Weiss, Multi-Agent Systems: An Introduction to Distributed Artificial Intelligence. Addison-wesley Reading, 1999, vol. 1.

R. Karam, R. Lin, B.A. Butler, and M. Egerstedt, "Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule", submitted to the 2025 IEEE Conference on Decision and Control (CDC).

Team-Level Collaboration Framework



Team-Level Collaboration Framework

Hamilton's Rule [1]	Hamilton's Rule for Team-Level Collaboration	
$rB \geq C$	$r_{ij}B_j > C_i, \quad (i, j) \in E$	
r : genetic relatedness	r_{ij} : relative mission importance	$r_{ij} = \frac{w_j}{w_i} \quad \text{and} \quad r_{ji} = \frac{w_i}{w_j} = \frac{1}{r_{ij}}$
B : benefit to the recipient	B_j : benefit to team j	$B_k = F_k(n_k + 1) - F_k(n_k)$
C : cost to the donor	C_i : cost to team i	$C_k = F_k(n_k) - F_k(n_k - 1)$

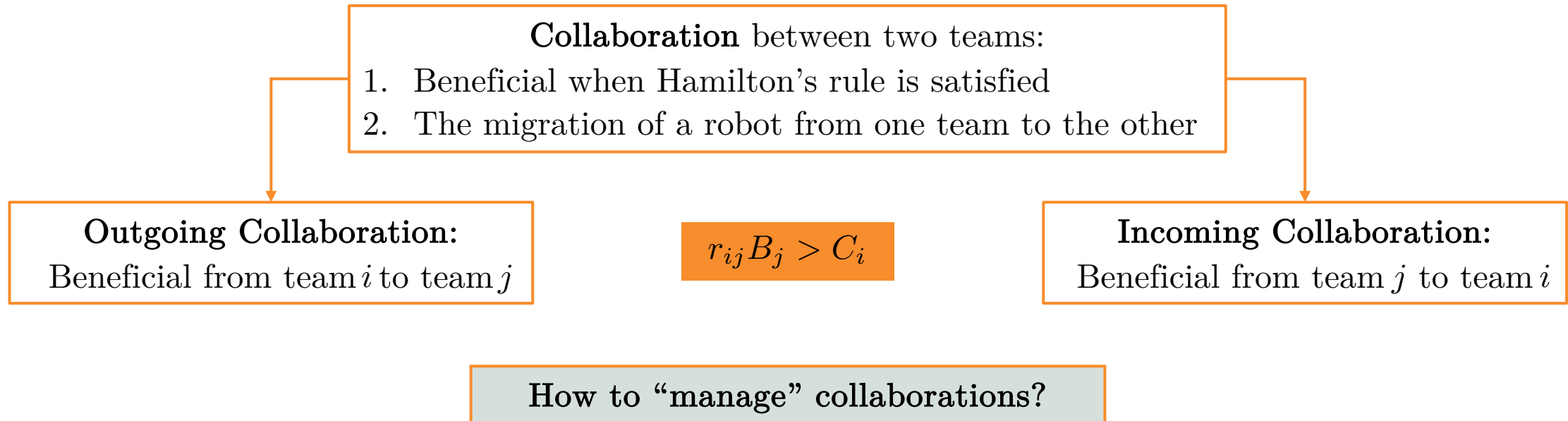
Hamilton's Rule for Team-Level Collaboration is defined as:

$$r_{ij}[F_j(n_j + 1) - F_j(n_j)] > [F_i(n_i) - F_i(n_i - 1)]$$

[1]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17–52, 1964.

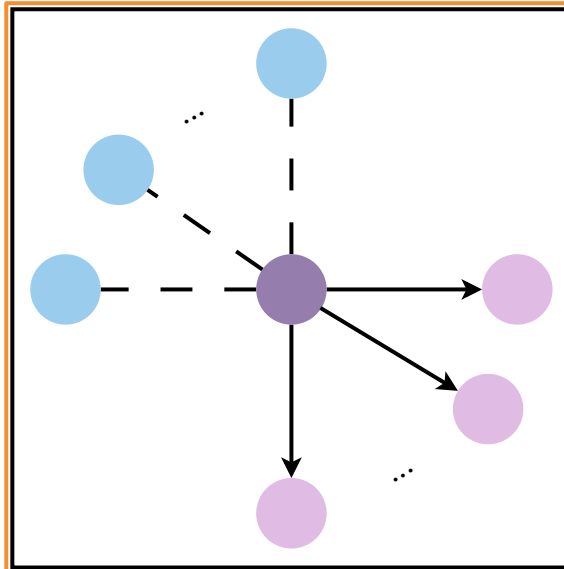
R. Karam, R. Lin, B.A. Butler, and M. Egerstedt, "Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule", submitted to the 2025 IEEE Conference on Decision and Control (CDC).

Team-Level Collaboration Framework



Theorem 1. Consider two teams i and j , with $i, j \in V$, each with at least one agent. Under Assumptions 1 and 2, if $r_{ij}B_j > C_i$, it follows that $r_{ji}B_i \leq C_j$.

Team-Level Collaboration Framework



Legend:

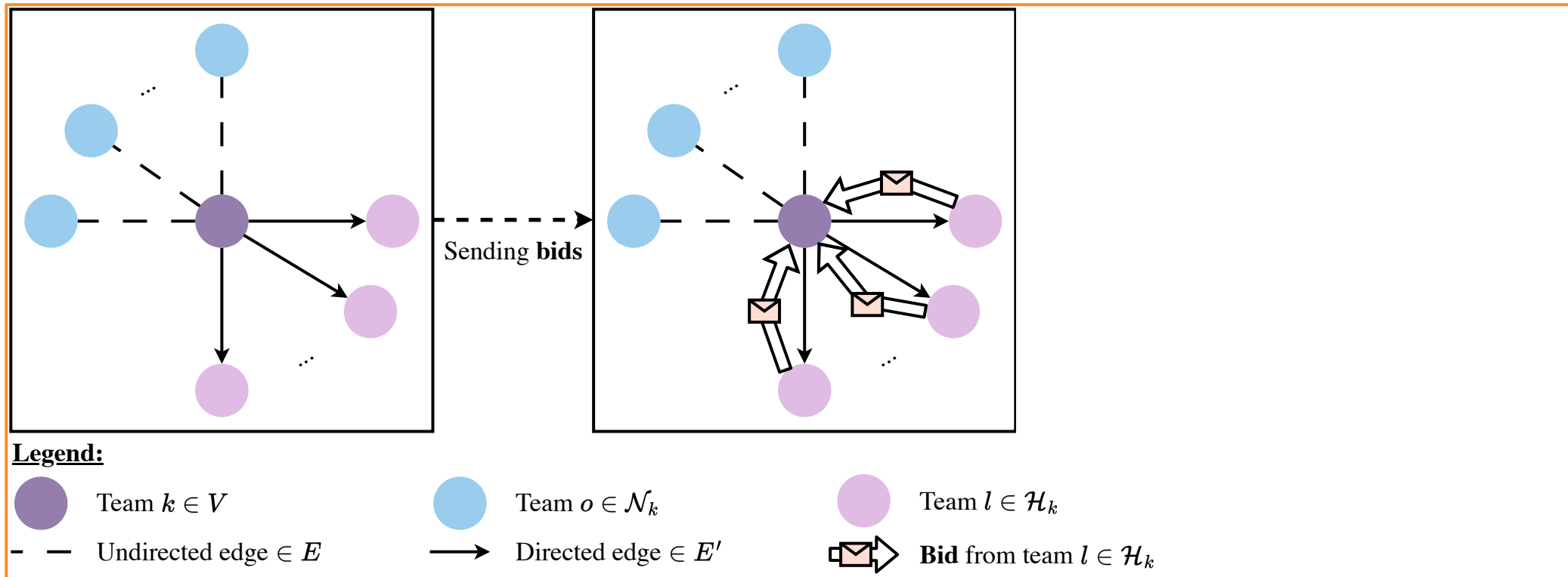
● Team $k \in V$
- - Undirected edge $\in E$

● Team $o \in \mathcal{N}_k$
→ Directed edge $\in E'$

● Team $l \in \mathcal{H}_k$

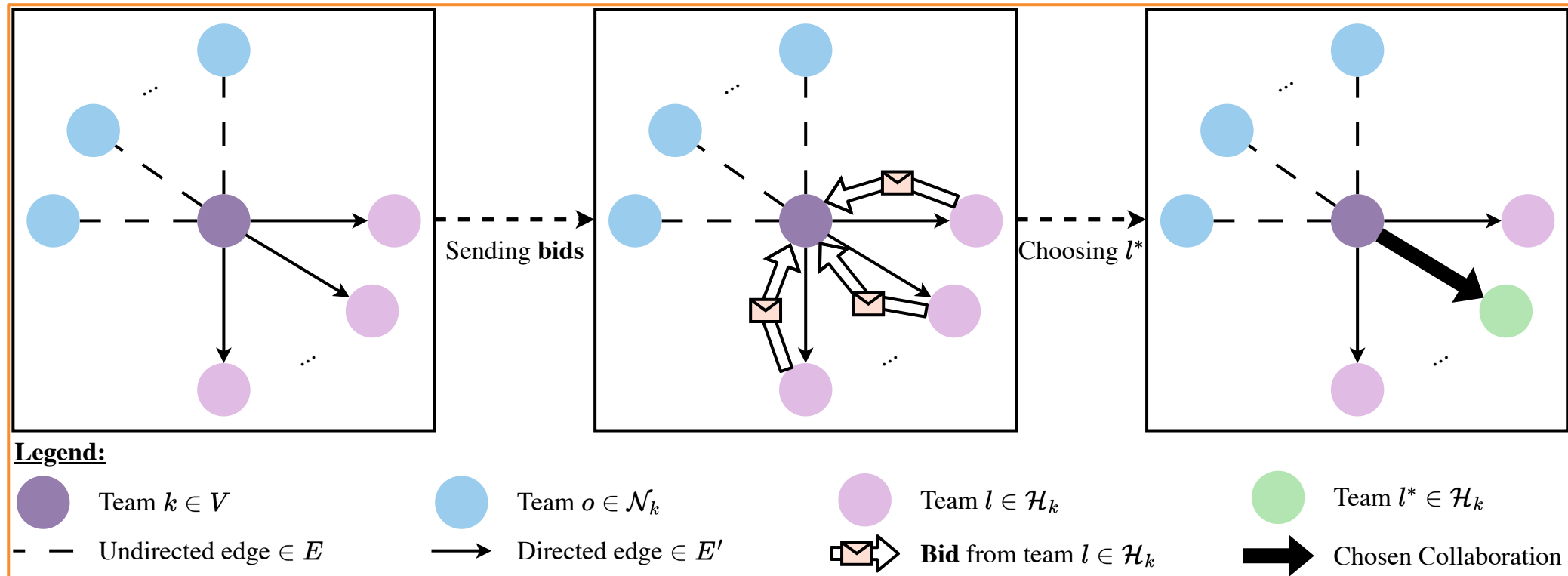
Outgoing Bidding Process for Team k

Team-Level Collaboration Framework



Outgoing Bidding Process for Team k

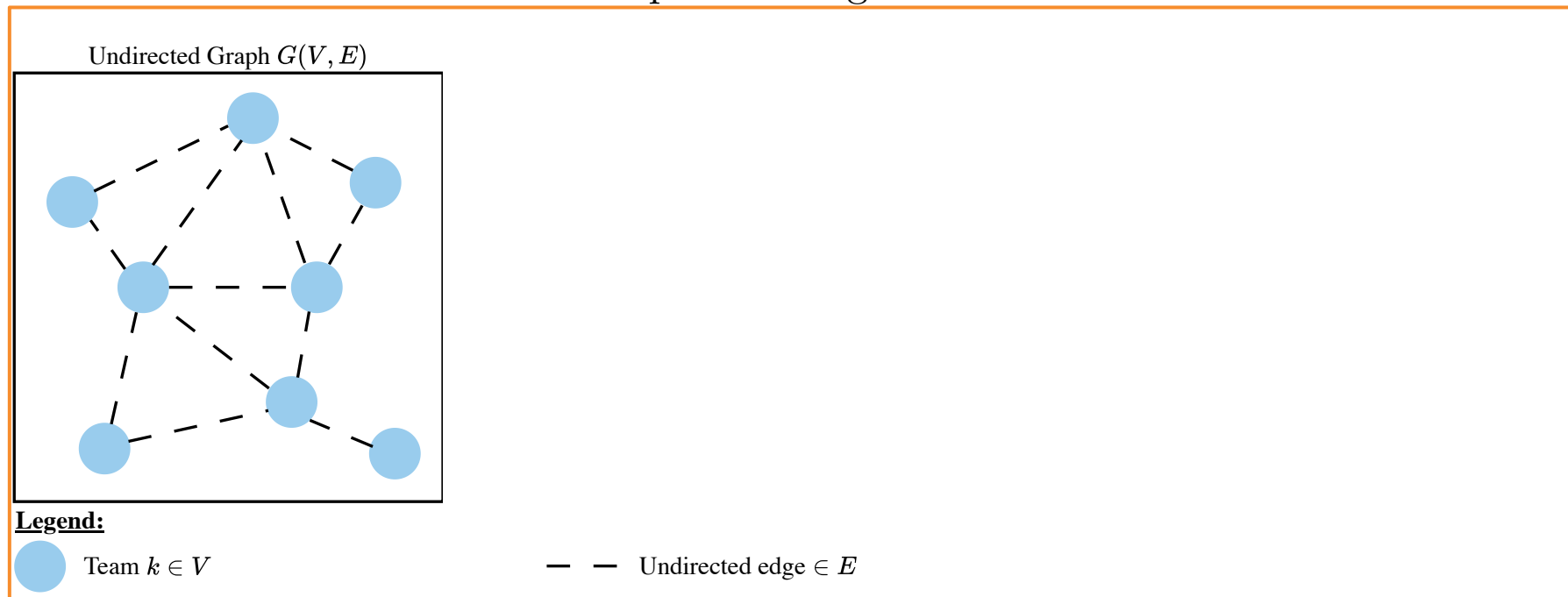
Team-Level Collaboration Framework



Outgoing Bidding Process for Team k

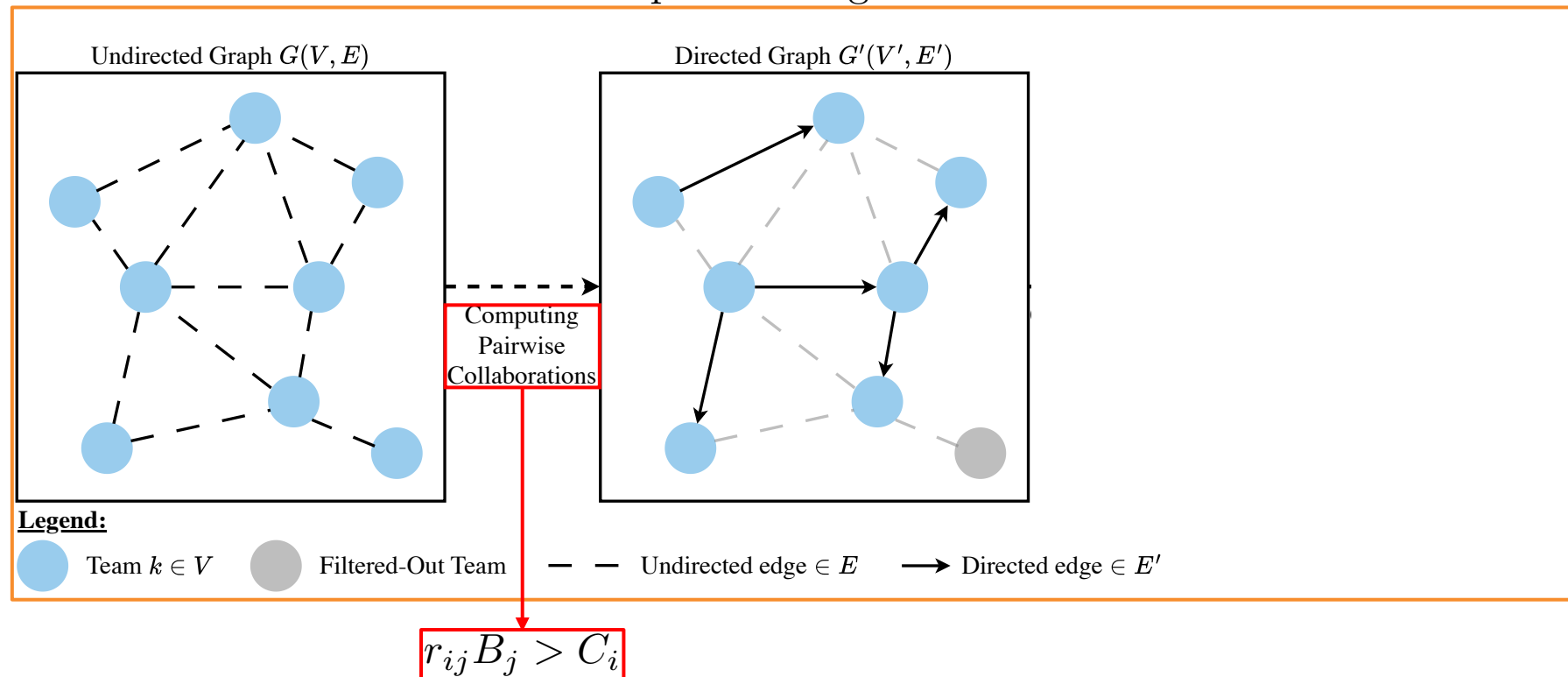
Team-Level Collaboration Framework

Graph Filtering Process



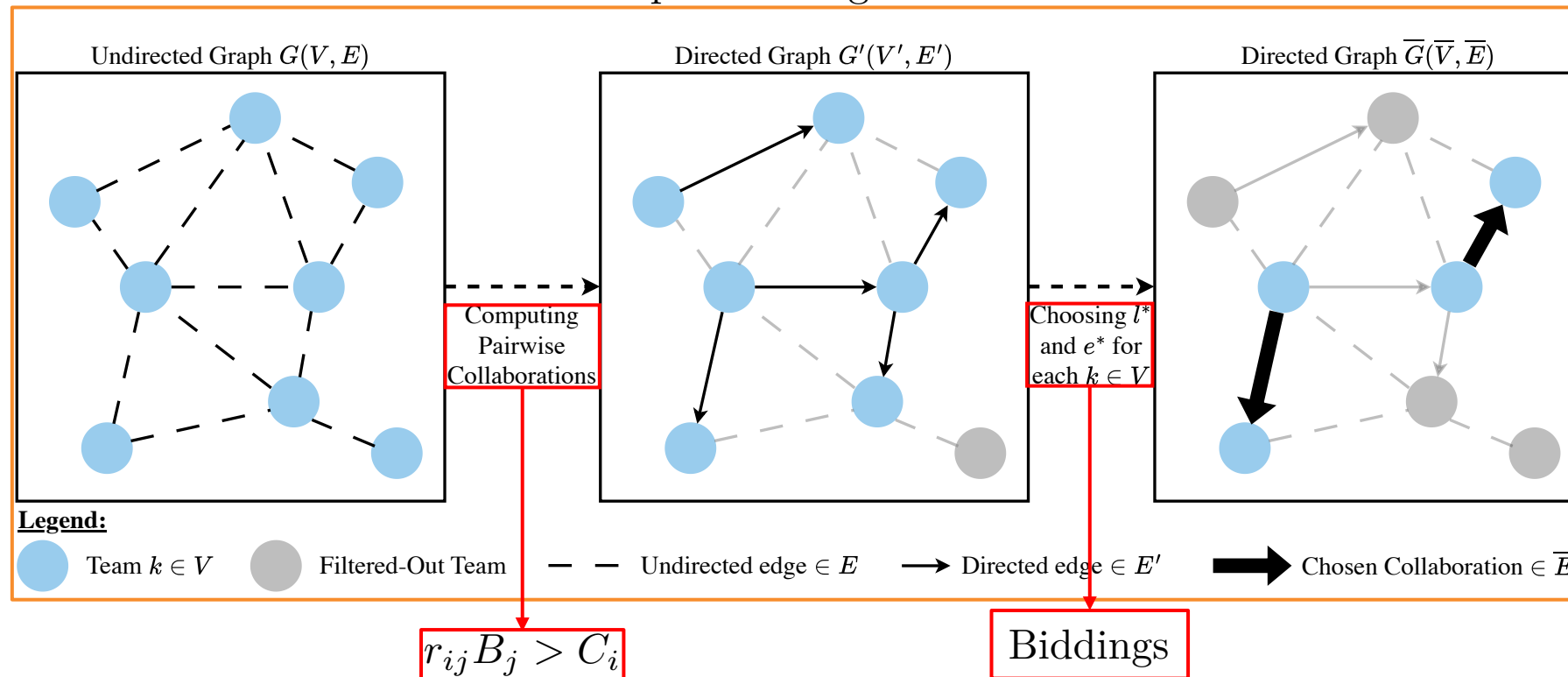
Team-Level Collaboration Framework

Graph Filtering Process



Team-Level Collaboration Framework

Graph Filtering Process



Team-Level Collaboration Framework

Global Mission Evaluation Function:

$$\mathcal{G}(n_1, \dots, n_m) = \sum_{k=1}^m w_k F_k(n_k)$$



Global Inequality Rule:

$$\mathcal{G}(n'_1, \dots, n'_m) > \mathcal{G}(n_1, \dots, n_m)$$



Global Benefit
instead of local
(or greedy)



Philosophy:

“If all of us benefit,
each of us benefits”



$\Omega = \left\{ (n_1, \dots, n_m) \in \mathbb{Z}_{>0}^m \mid \sum_{k=1}^m n_k = N \right\}$: set of all allocations of
the m agents between the N teams

More formally,

$$\mathcal{G} : \Omega \rightarrow \mathbb{R}$$

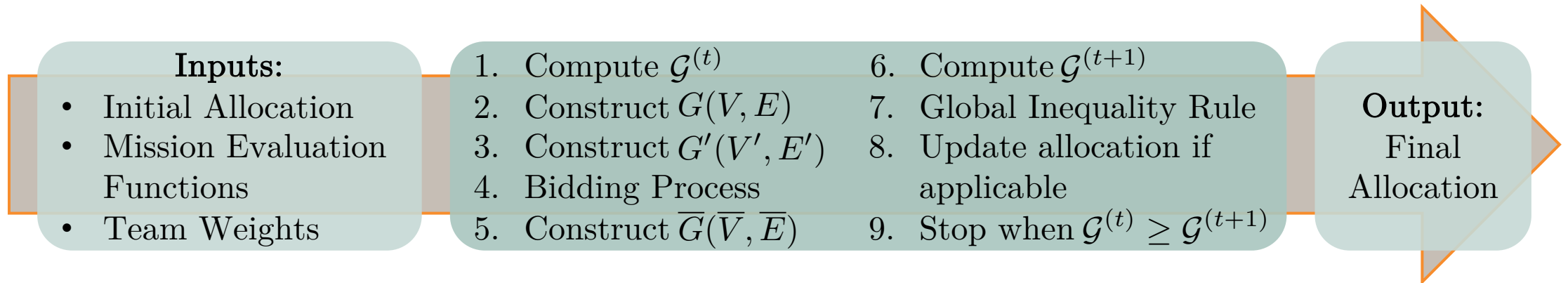
N total number of robots across the m teams

Team-Level Collaboration Framework

Lemma 1. There exists an allocation $(n_1^*, \dots, n_m^*) \in \Omega$, such that

$$\mathcal{G}(n_1^*, \dots, n_m^*) \geq \mathcal{G}(n_1, \dots, n_m) \quad \forall (n_1, \dots, n_m) \in \Omega$$

In other words, $\mathcal{G}$ attains a maximum value on Ω , and $(n_1^*, \dots, n_m^*)$ is an optimal allocation.



Theorem 2. This iterative process terminates in a finite number of steps and converges to an allocation $(n_1^*, \dots, n_m^*) \in \Omega$ that maximizes $\mathcal{G}$.

Team Mission: Area Coverage

Team Mission



Optimal Area Coverage



Voronoi-Based Coverage Control [1]

$N \in \mathbb{Z}^+$ Robots



Convex and Compact $\mathcal{D} \subset \mathbb{R}^d$



$$L(p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i} \|p_i - q\|^2 \phi(q) dq$$



Dynamic Area
Coverage with
Fire-Fighting
Robots [2]

Centroidal Voronoi Tessellation (CVT)

$$p_i = c_i = \frac{\int_{\mathcal{V}_i} q \phi(q) dq}{\int_{\mathcal{V}_i} \phi(q) dq}, \quad \forall i \in \mathcal{N}$$

- Centroid: $c_i \in \mathbb{R}^d$
- CVT: $c = [c_1^\top, c_2^\top, \dots, c_N^\top]^\top$

[1] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," IEEE Transactions on Robotics and Automation, vol. 20, no. 2, pp. 243–255, 2004.

[2]: R. Lin, S. Kim, and M. Egerstedt, "Heterogeneous Collaborative Pursuit via Coverage Control Driven by Fokker–Planck Equations," IEEE Transactions on Robotics (T-RO), 2025.

R. Karam, R. Lin, B.A. Butler, and M. Egerstedt, "Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule", submitted to the 2025 IEEE Conference on Decision and Control (CDC).

Team Mission: Area Coverage

Mission Evaluation Function: $F(N) = -L(N, c)$, with: $L(N, p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i} \|p_i - q\|^2 \phi(q) dq$

Assumption 2:
Mission Evaluation Function Properties

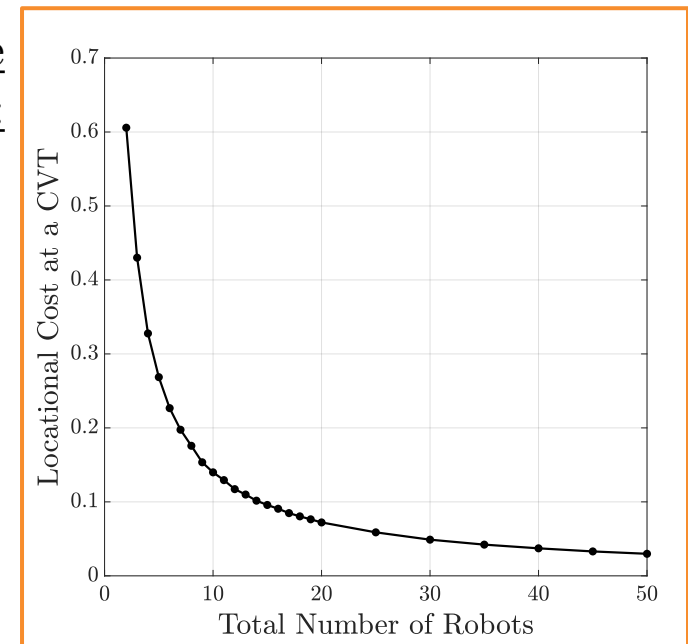
Discrete
Convexity:

Strictly Decreasing:

Theorem 3. Suppose $p_i \in \mathcal{D}$, $\forall i \in \mathcal{N}$. By adding one more robot $p_{N+1} \in \mathcal{D} \setminus \{p_i\}$, $\forall i \in \mathcal{N}$, the locational cost decreases, i.e.,

$$L(N + 1, p') < L(N, p), \quad \forall N \in \mathbb{Z}^+$$

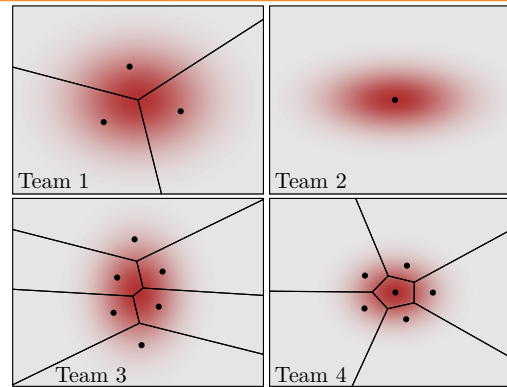
where $p' = [p^\top, p_{N+1}^\top]^\top$.



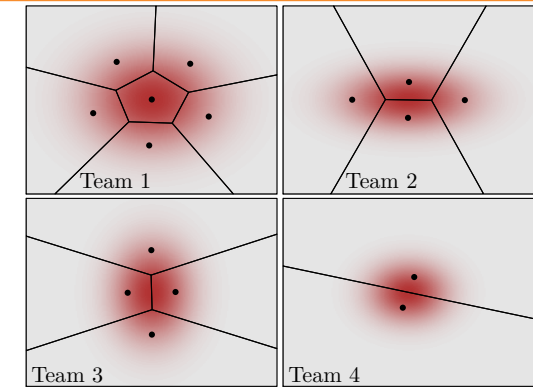
Simulations

Case 1:

- Same mission importances
- Different density functions



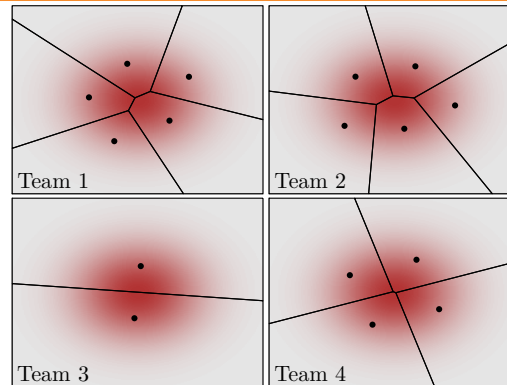
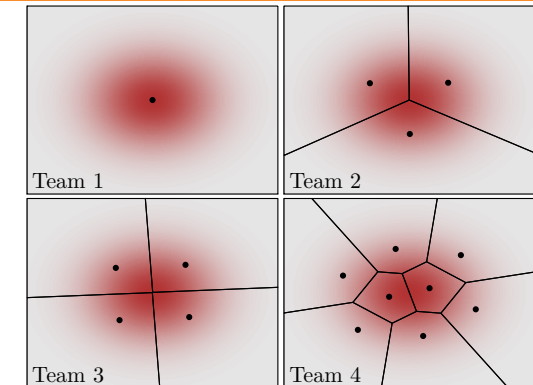
Initial Allocation



Final Allocation

Case 2:

- Different mission importances
- Same density functions



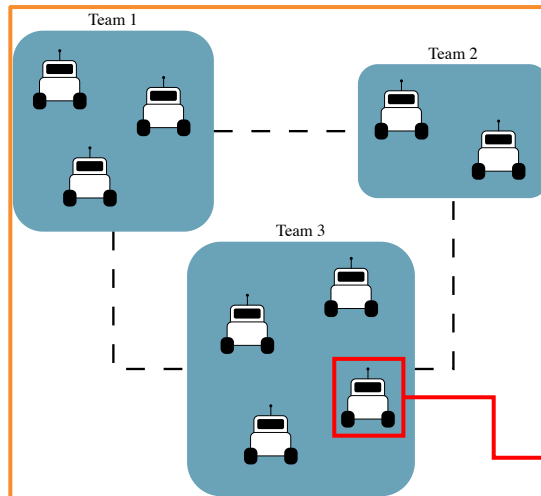
Conclusion

- Ecologically-Inspired Framework: Hamilton's Rule
- Multi-Team Environments
- Optimal Resource Allocation
- Robots Considered as Resources



Framework Achieves Optimal Performance for the Entire System through Altruistic Behavior

Thank you for listening!



Hamilton's Rule for
Team-Level
Collaboration:

$$r_{ij}[F_j(n_j + 1) - F_j(n_j)] > [F_i(n_i) - F_i(n_i - 1)]$$

Robot = Resource

