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Resource Allocation with Multi-Team Collaboration Based on Hamilton's Rule

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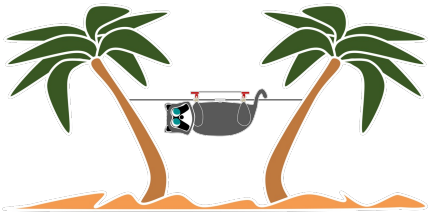


Motivation

Los Angeles County fires: Maps of where the wildfires are [1]

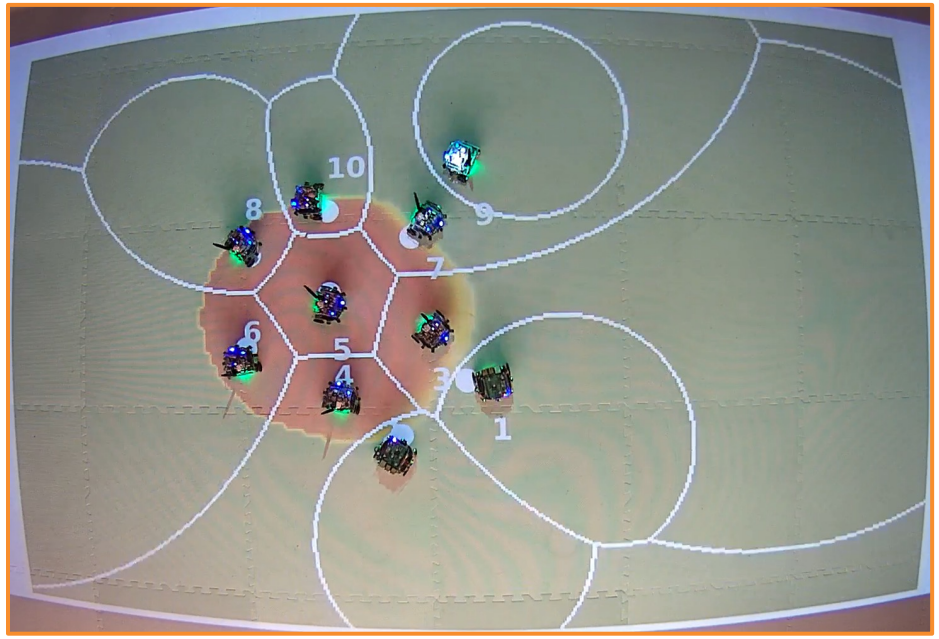
Observations:

- 4 Distinct Regions of Interest
- Each has a certain fire intensity
- Each has a certain population density

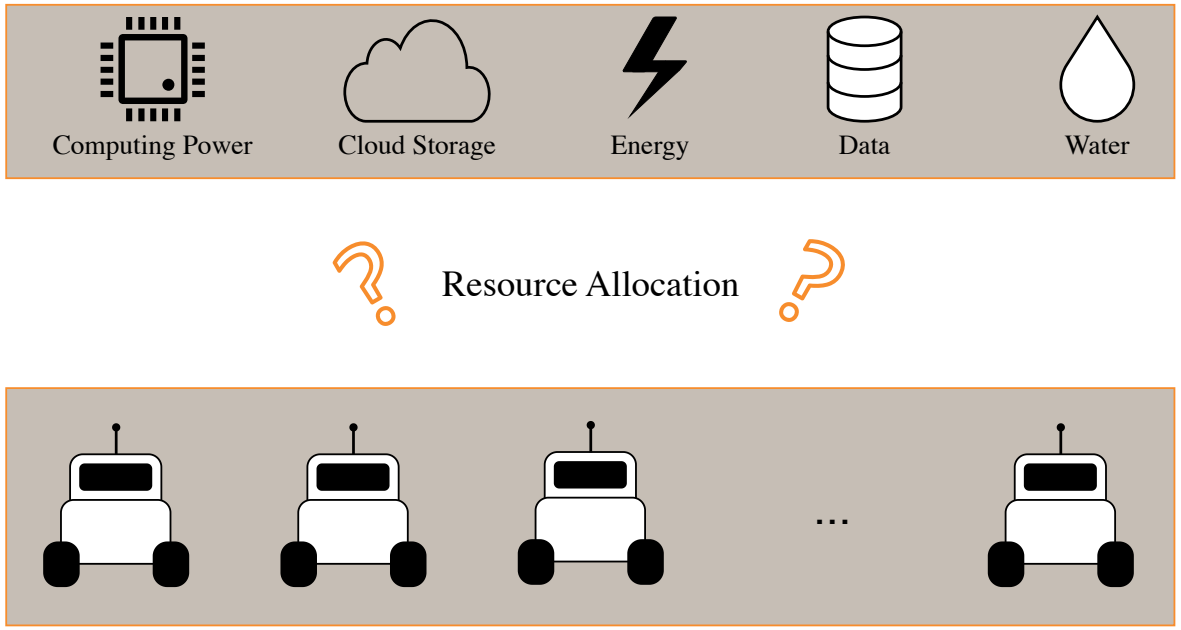
Multi-Robotic Solution?

[1]: https://www.abc10.com/video/news/local/california/los-angeles-county-fires-maps-of-where-the-wildfires-are/103-e4c8ed38-68b4-4e7a-bfca-50073bfbfd3f

Previous Literature & Resource Allocation



Dynamic Area Coverage with Fire-Fighting Robots [1]



Resource Allocation in Multi-Agent Systems, e.g., [2]

Q1. How to implement resource allocation for such scenarios?

[1]: R. Lin, S. Kim, and M. Egerstedt, "Heterogeneous Collaborative Pursuit via Coverage Control Driven by Fokker–Planck Equations," IEEE Transactions on Robotics (T-RO), 2025.
[2]: P. Anussornnitisarn, S. Y. Nof, and O. Etzion, "Decentralized control of cooperative and autonomous agents for solving the distributed resource allocation problem," International Journal of Production Economics, vol. 98, no. 2, pp. 114–128, 2005.

Ecological Inspiration

Hamilton's Rule of Altruism: [1]

$$rB \geq C$$

- r : genetic relatedness
- B : benefit to the recipient
- C : cost to the donor

Lioness adopts deceased sister's cubs [2]

- Genetic relatedness: sibling relationship, lioness and her deceased sister
- Benefit to the **lion cubs' deceased mother:** her genes will survive through her cubs' survival
- Cost to the **lioness:** sharing her lion cubs' resources and splitting her care.

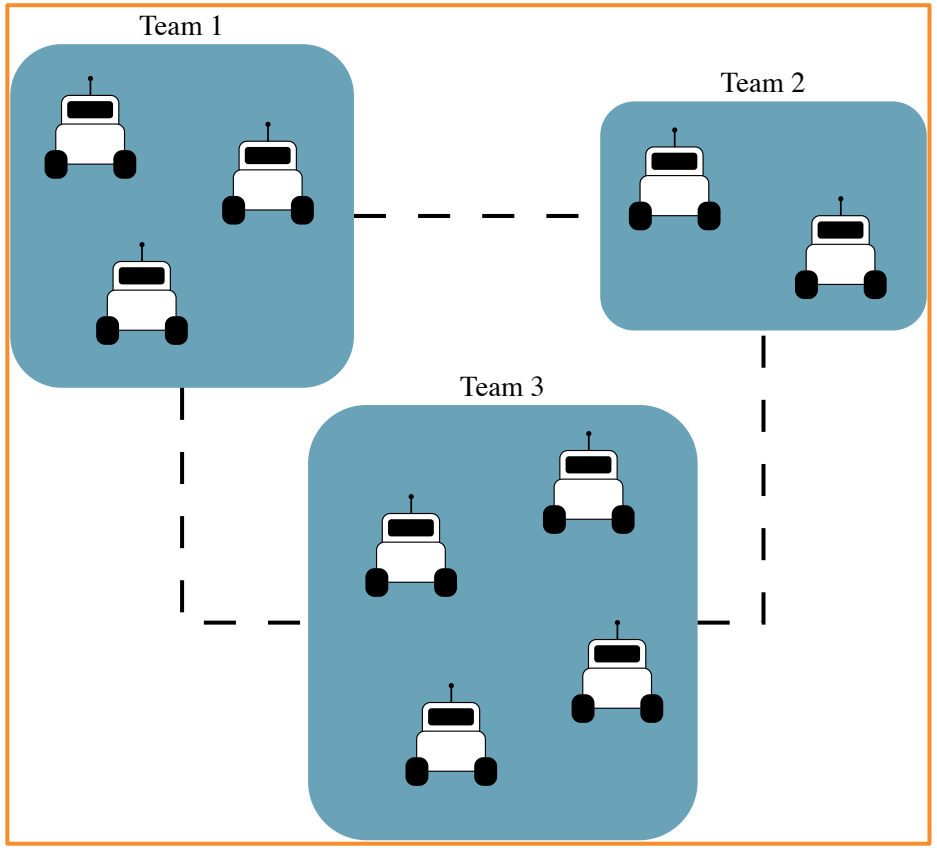
Q2. How to translate Hamilton's rule to robotic systems?

Lioness → Team of Robots

Genes → Mission Weights

[1]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17–52, 1964.
[2]: https://www.dailymail.co.uk/news/article-4418844/Lioness-suckles-orphaned-cubs-adopted-sister.html

Collaboration: Robots as Resources



Graph Modeling: [1]

- Let there be m teams of robots
- $G = (V, E)$: undirected graph
- $V = \{v_1, \ldots, v_m\}$: set of **teams**
- $E \subseteq V \times V$: set of **potential collaborative interactions**

Key Idea:

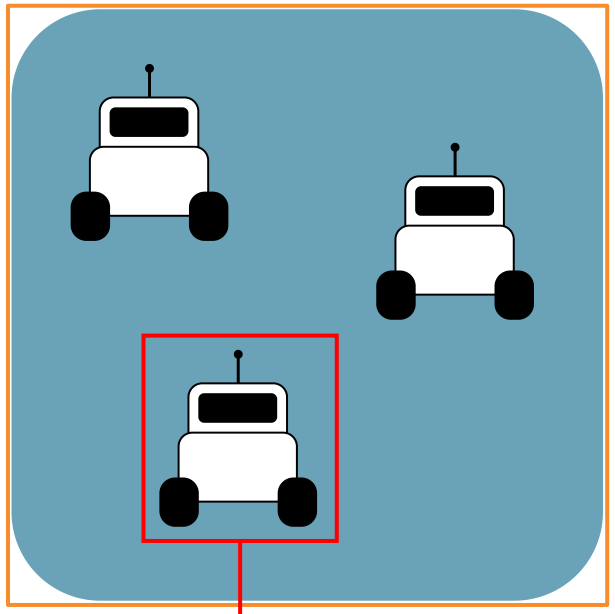
Compute potential collaborative interactions inspired by Hamilton's Rule [2]

Assumption 1:

All agents in all m teams are homogenous, i.e., identical

[1]: M. Mesbahi and M. Egerstedt, Graph Theoretic Methods in Multiagent Networks. Princeton University Press, 2010.
[2]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17–52, 1964.

Team-Level Collaboration Framework



Robot = Resource

How to define a team's mission performance?

Mission Evaluation Function

Which parameter(s) to consider?

Number of Robots in a Team

Mission Evaluation Function for team k: $F_k(n_k)$,
with n_k: number of robots in team k

Team-Level Collaboration Framework

Mission Evaluation Function for team k: $F_k(n_k)$, with n_k: number of robots in team k

Assumption 2:

For each team k, the mission evaluation function $F_k(n_k) : \mathbb{N} \to \mathbb{R}$ satisfies the following properties:

1. Strictly Increasing: For all $n \geq 1$,
$$F_k(n+1) > F_k(n)$$
2. Discrete Concavity: For all $n \geq 2$,
$$F_k(n+1) - F_k(n) \leq F_k(n) - F_k(n-1)$$

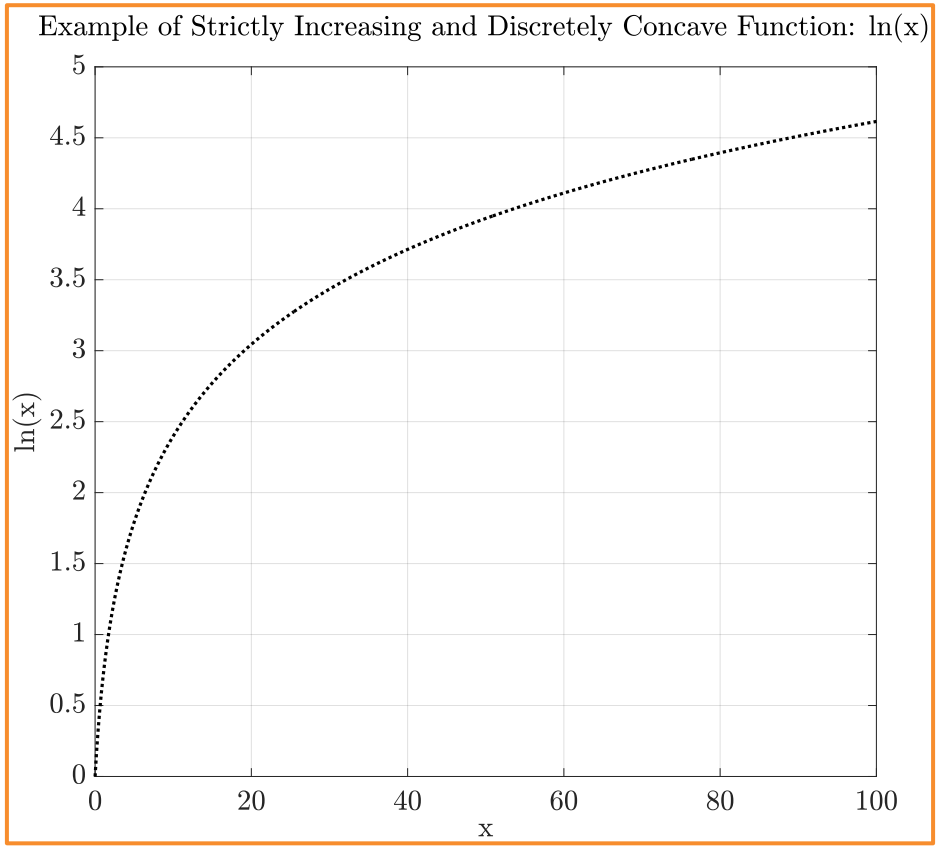
Higher is better

Diminishing Returns

Scenario: [1]
There won't be overcrowding of agents and the number of agents in each team at any point in time won't reach a point where decreasing returns occur.

[1]: J. Ferber and G. Weiss, Multi-Agent Systems: An Introduction to Distributed Artificial Intelligence. Addison-wesley Reading, 1999, vol. 1.

Team-Level Collaboration Framework



Team-Level Collaboration Framework

Hamilton's Rule [1]	Hamilton's Rule for Team-Level Collaboration	
$rB \geq C$	$r_{ij}B_j > C_i, \quad (i,j) \in E$	
r: genetic relatedness	r_{ij}: relative mission importance	$r_{ij} = \frac{w_j}{w_i} \quad and \quad r_{ji} = \frac{w_i}{w_j} = \frac{1}{r_{ji}}$
B: benefit to the recipient	B_j: benefit to team j	$B_k = F_k(n_k + 1) - F_k(n_k)$
C: cost to the donor	C_i: cost to team i	$C_k = F_k(n_k) - F_k(n_k - 1)$

Hamilton's Rule for Team-Level Collaboration is defined as:

$$r_{ij}[F_j(n_j + 1) - F_j(n_j)] > [F_i(n_i) - F_i(n_i - 1)]$$

[1]: W. D. Hamilton, "The genetical evolution of social behaviour. ii," Journal of Theoretical Biology, vol. 7, no. 1, pp. 17–52, 1964.

Team-Level Collaboration Framework

Collaboration between two teams:

1. Beneficial when Hamilton's rule is satisfied
2. The migration of a robot from one team to the other

Outgoing Collaboration:
Beneficial from team i to team j

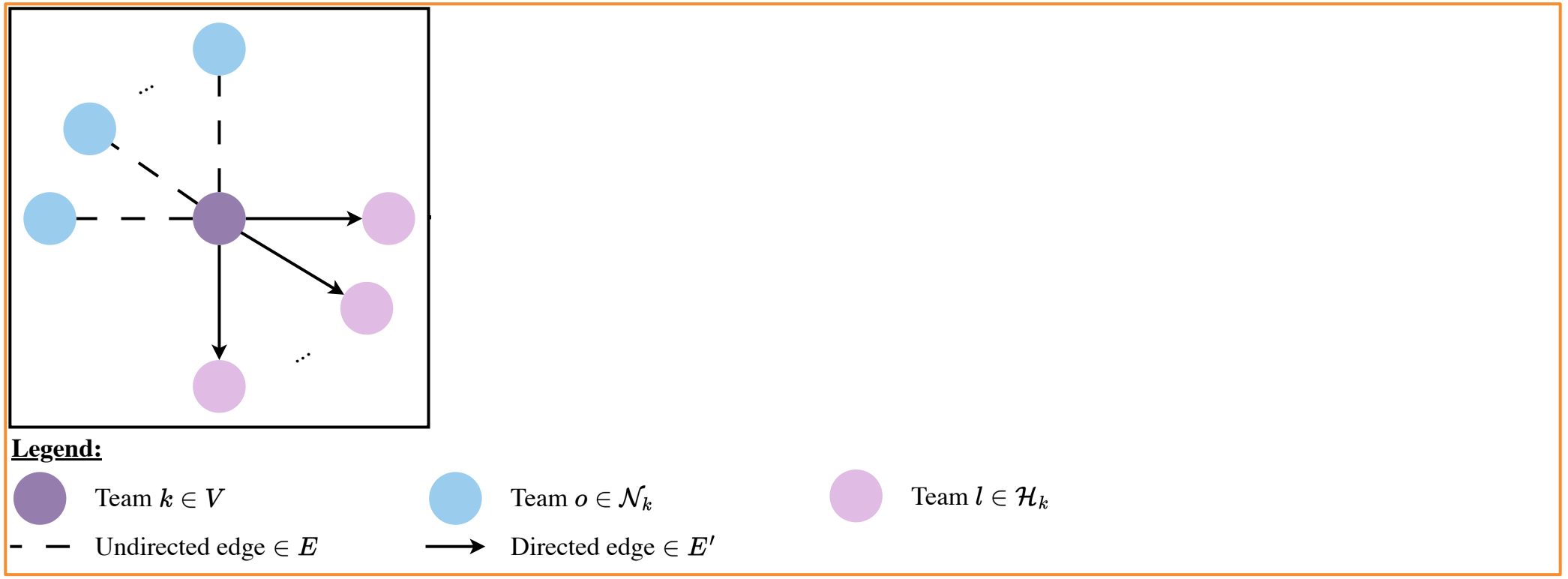
$$r_{ij} B_j > C_i$$

Incoming Collaboration:
Beneficial from team j to team i

How to "manage" collaborations?

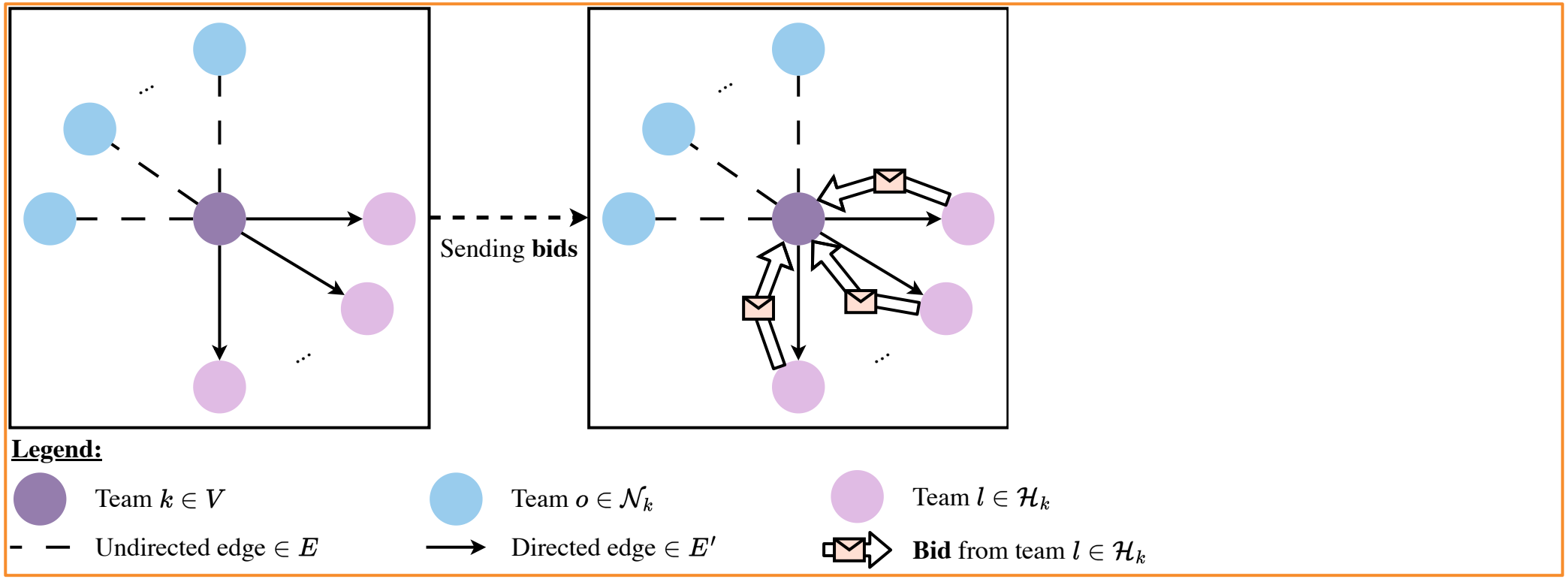
Theorem 1. Consider two teams i and j, with $i, j \in V$, each with at least one agent. Under Assumptions 1 and 2, if $r_{ij} B_j > C_i$, it follows that $r_{ji} B_i \leq C_j$.

Team-Level Collaboration Framework



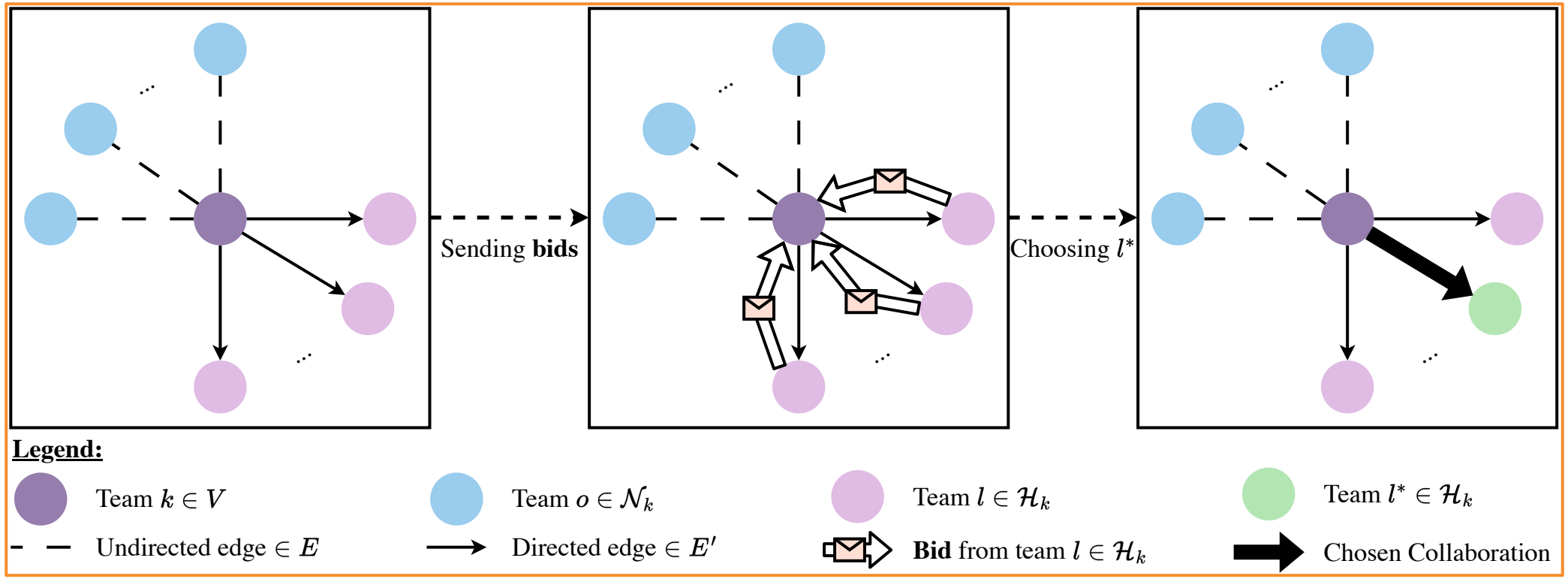
Outgoing Bidding Process for Team k

Team-Level Collaboration Framework



Outgoing Bidding Process for Team k

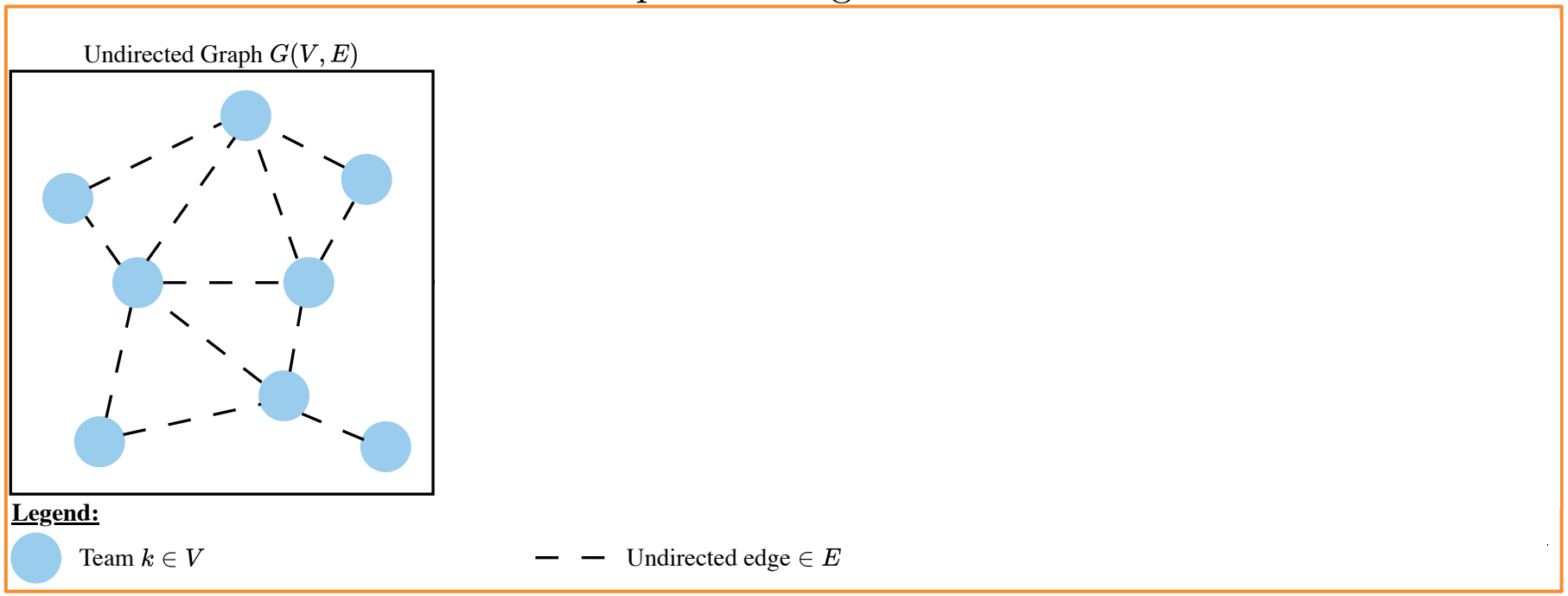
Team-Level Collaboration Framework



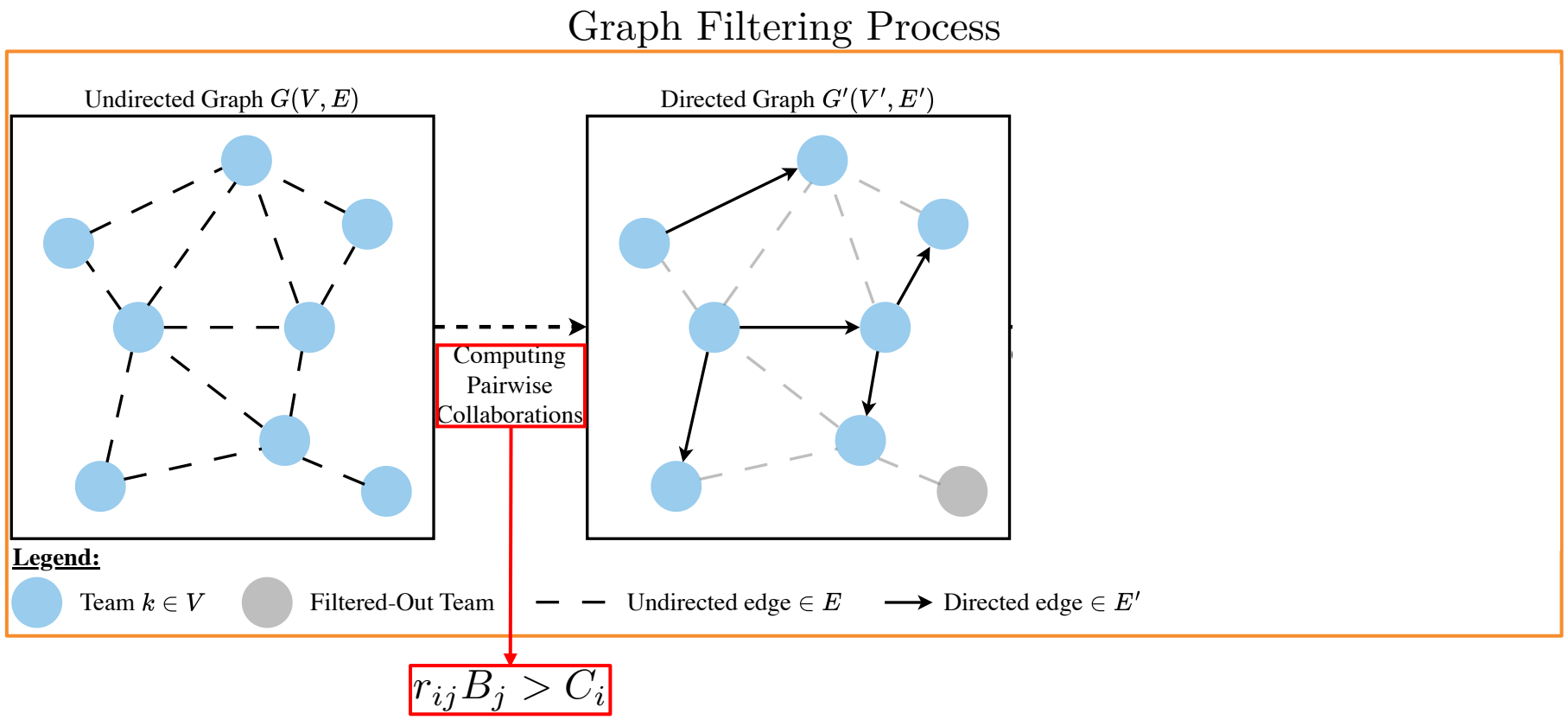
Outgoing Bidding Process for Team k

Team-Level Collaboration Framework

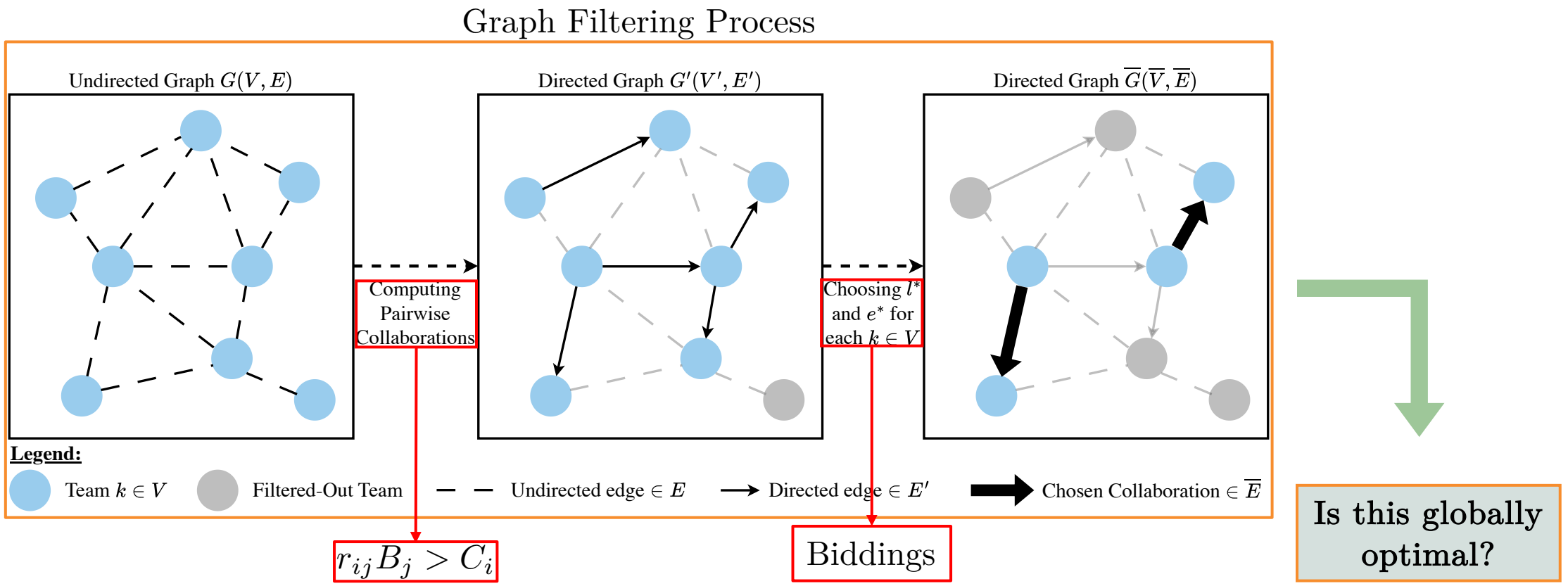
Graph Filtering Process



Team-Level Collaboration Framework



Team-Level Collaboration Framework



Team-Level Collaboration Framework

Global Mission Evaluation Function:

$$\mathcal{G}(n_1, \ldots, n_m) = \sum_{k=1}^{m} w_k \, F_k(n_k)$$

N total number of robots across the m teams

Global Inequality Rule:

$$\mathcal{G}(n'_1, \ldots, n'_m) > \mathcal{G}(n_1, \ldots, n_m)$$

→ Global Benefit instead of local (or greedy)

→ **Philosophy:** "If all of us benefit, each of us benefits"

$\Omega = \left\{ (n_1, \ldots, n_m) \in \mathbb{Z}^m_{>0} \;\middle|\; \sum_{k=1}^{m} n_k = N \right\}$: **set of all allocations of the N agents between the m teams**

→ More formally,

$$\mathcal{G} : \Omega \to \mathbb{R}$$

Team-Level Collaboration Framework

Lemma 1. There exists an allocation $(n_1^*, \ldots, n_m^*) \in \Omega$, such that

$$\mathcal{G}(n_1^*, \ldots, n_m^*) \geq \mathcal{G}(n_1, \ldots, n_m) \quad \forall (n_1, \ldots, n_m) \in \Omega$$

In other words, $\mathcal{G}$ attains a maximum value on Ω, and $(n_1^*, \ldots, n_m^*)$ is an optimal allocation.

Inputs:

- Initial Allocation
- Mission Evaluation Functions
- Team Weights

1. Compute $\mathcal{G}^{(t)}$
2. Construct $G(V, E)$
3. Construct $G'(V', E')$
4. Bidding Process
5. Construct $\overline{G}(\overline{V}, \overline{E})$
6. Compute $\mathcal{G}^{(t+1)}$
7. Global Inequality Rule
8. Update allocation if applicable
9. Stop when $\mathcal{G}^{(t)} \geq \mathcal{G}^{(t+1)}$

Output: Final Allocation

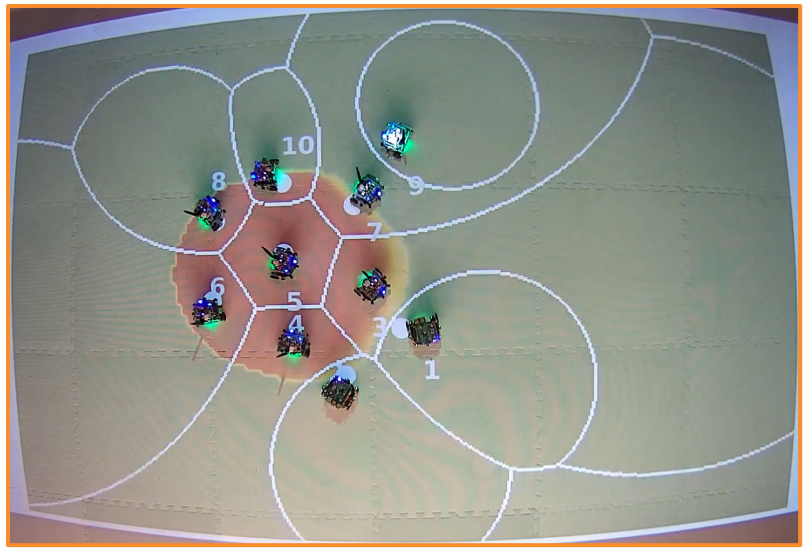
Theorem 2. This iterative process terminates in a finite number of steps and converges to an allocation $(n_1^*, \ldots, n_m^*) \in \Omega$ that maximizes $\mathcal{G}$.

Team Mission: Area Coverage

Team Mission → Optimal Area Coverage → Voronoi-Based Coverage Control [1]

$N \in \mathbb{Z}^+$ Robots → Convex and Compact $\mathcal{D} \subset \mathbb{R}^d$ →

$$L(p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i} \|p_i - q\|^2 \phi(q)\, dq$$



Dynamic Area Coverage with Fire-Fighting Robots [2]

Centroidal Voronoi Tessellation (CVT)

$$p_i = c_i = \frac{\int_{\mathcal{V}_i} q\phi(q)\, dq}{\int_{\mathcal{V}_i} \phi(q)\, dq}, \quad \forall i \in \mathcal{N}$$

- Centroid: $c_i \in \mathbb{R}^d$
- CVT: $c = [c_1^\top, c_2^\top, \ldots, c_N^\top]^\top$

[1] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," IEEE Transactions on Robotics and Automation, vol. 20, no. 2, pp. 243–255, 2004.
[2]: R. Lin, S. Kim, and M. Egerstedt, "Heterogeneous Collaborative Pursuit via Coverage Control Driven by Fokker–Planck Equations," IEEE Transactions on Robotics (T-RO), 2025.

Team Mission: Area Coverage

Mission Evaluation Function: $F(N) = -L(N, c)$, **with:** $L(N, p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i} \|p_i - q\|^2 \phi(q)\, dq$

Assumption 2:
Mission Evaluation Function Properties

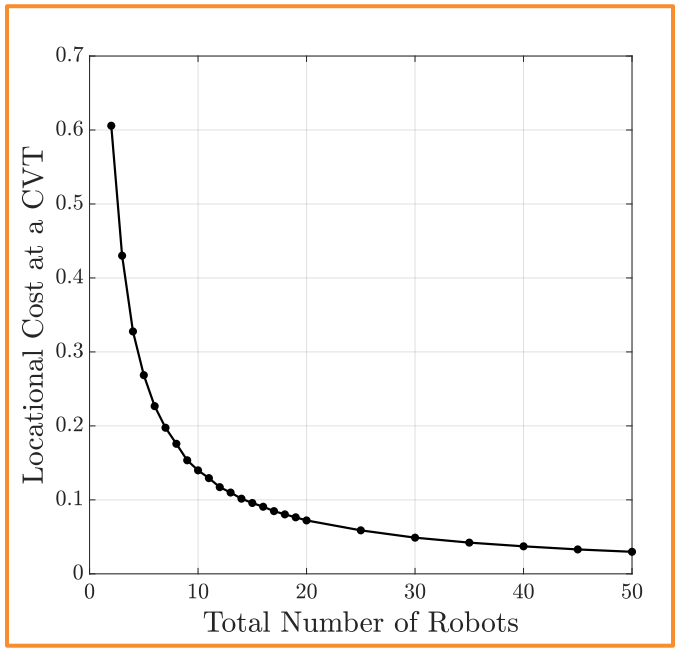
→ Discrete Convexity:

→ Strictly Decreasing:

Theorem 3. Suppose $p_i \in \mathcal{D}$, $\forall i \in \mathcal{N}$. By adding one more robot $p_{N+1} \in \mathcal{D} \setminus \{p_i\}$, $\forall i \in \mathcal{N}$, the locational cost decreases, i.e.,

$$L(N+1, p') < L(N, p), \quad \forall N \in \mathbb{Z}^+$$

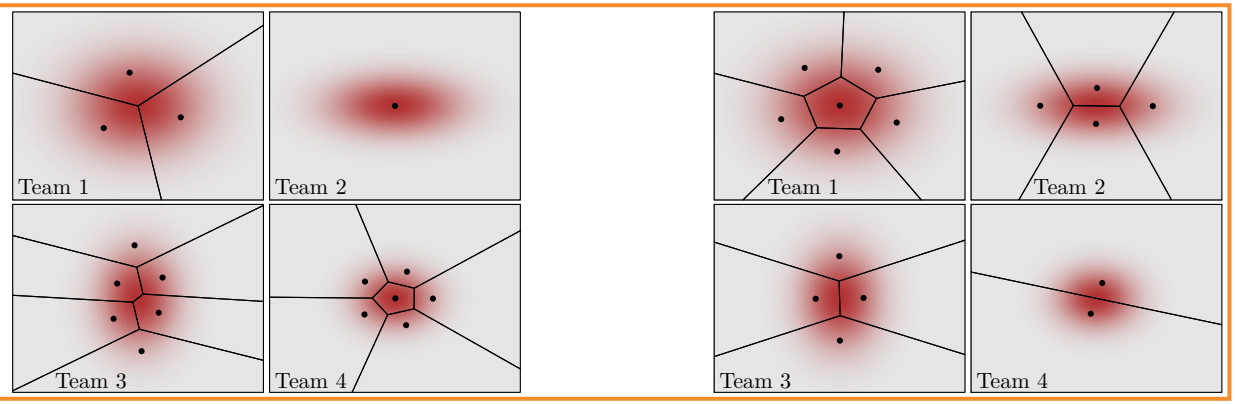
where $p' = [p^\top, p_{N+1}^\top]^\top$.



Simulations

Case 1:

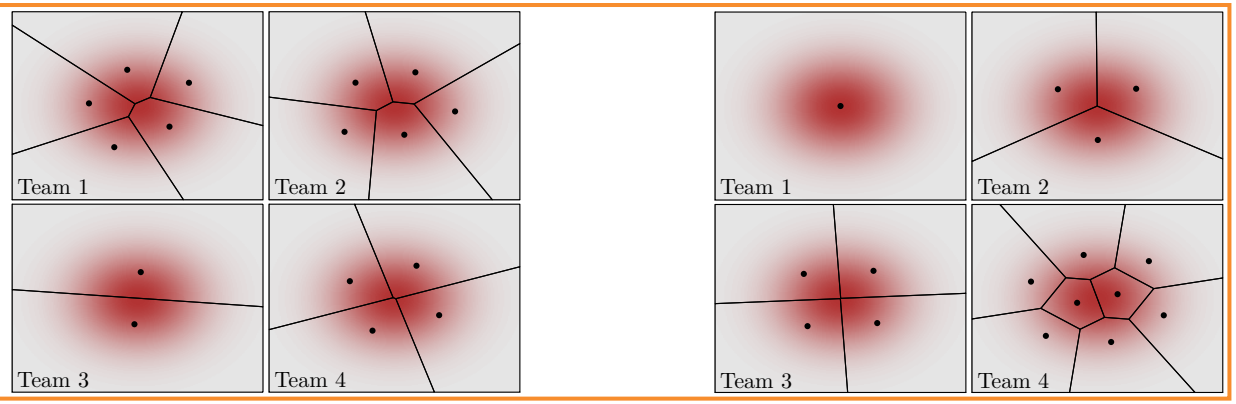
- **Same** mission importances
- **Different** density functions



Initial Allocation | Final Allocation

Case 2:

- **Different** mission importances
- **Same** density functions

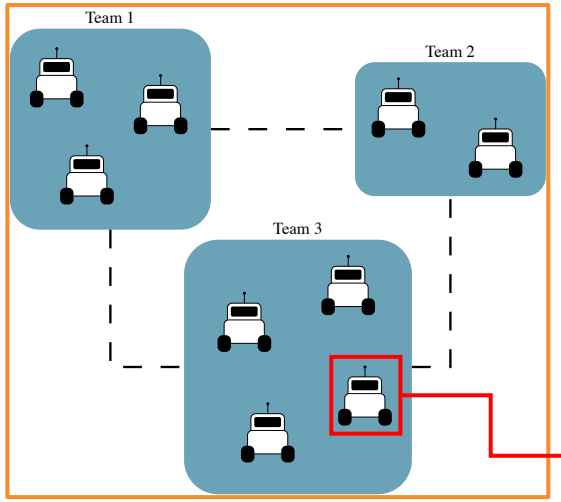


Conclusion

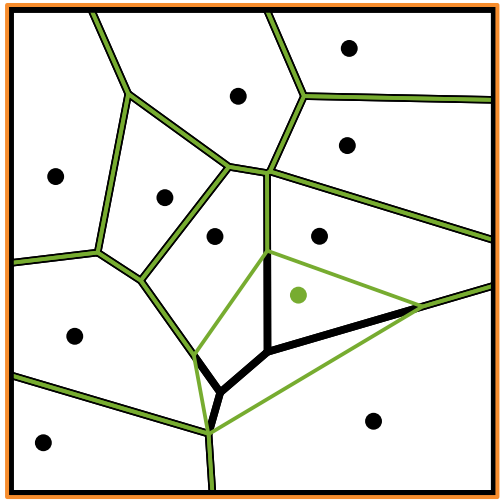
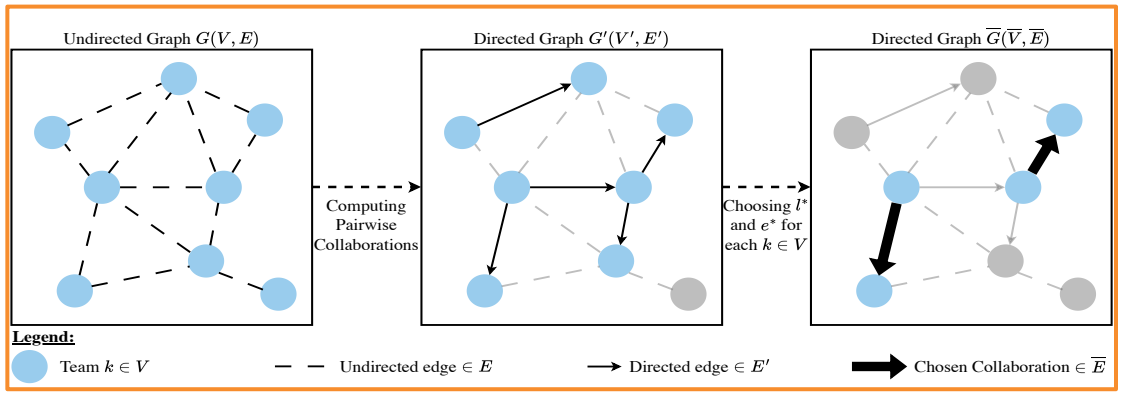
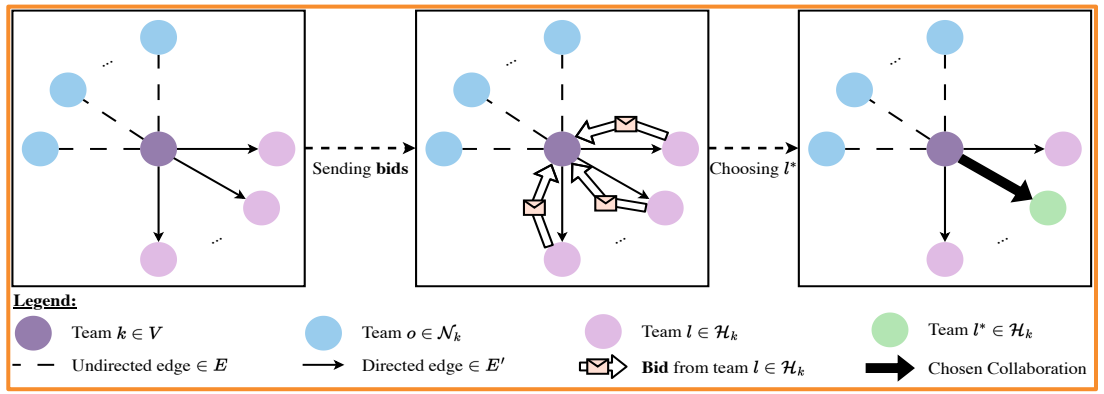
- Ecologically-Inspired Framework: Hamilton's Rule
- Multi-Team Environments
- Optimal Resource Allocation
- Robots Considered as Resources

Framework Achieves Optimal Performance for the Entire System through Altruistic Behavior

Thank you for listening!



Hamilton's Rule for Team-Level Collaboration:

$$r_{ij}[F_j(n_j+1) - F_j(n_j)] > [F_i(n_i) - F_i(n_i-1)]$$

Robot = Resource